Periodic Structures for Microwave Engineering

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ABSTRACT

Periodic structures have been extensively used in the history of microwave engineering. Recently, there appears a significant renewed interest in this subject, due to highly publicized Photonic Crystals and Metamaterials. This lecture introduces basic theory on the electromagnetic aspects and circuit representations of periodic structures. Several practical examples are introduced. Some fundamental features of potential use of the periodic structures are discussed.

INTRODUCTION

Periodic structures appear in nature in such forms as bee hives and crystals. They can be man-made such as the structures discussed in this lecture. Let us first define the periodic structure as discussed in this lecture. Periodic structure is made of an infinite or finite repetition of a unit cell in one, two or three dimensions. For instance, Fig.1 provides a simple example of the periodic repetition of a lumped element placed along a transmission line.

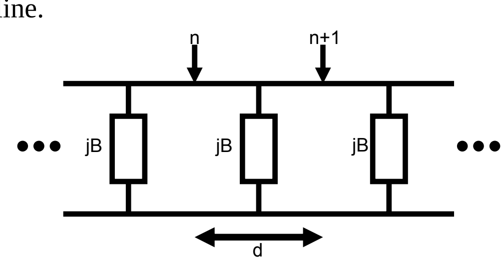

Fig.1 Transmission line loaded periodically with lumped elements

The structure above is a one-dimensional periodic structure. It is possible to obtain a two-dimensional periodic structure or possibly a three-dimensional one. However, in this lecture, the one-dimensional cases are primarily treated because the simpler structure is more useful for understanding.

EXAMPLE STRUCTURES

Fig.2 presents some of practical periodic structures used in microwaves as well as optics. (a) is a two dimensional periodic structure and the structure is open to support certain radiation properties. (b) is a closed structure so that no radiation is involved. Only interesting guided wave phenomena are observed. (c) is one of the most interesting periodic structures. This structure is made of periodic perturbations applied to an open (dielectric) waveguide. Therefore, both guided wave phenomena and radiation phenomena exist. If the structure is scaled to optical frequency, this type of structure is called Grating.

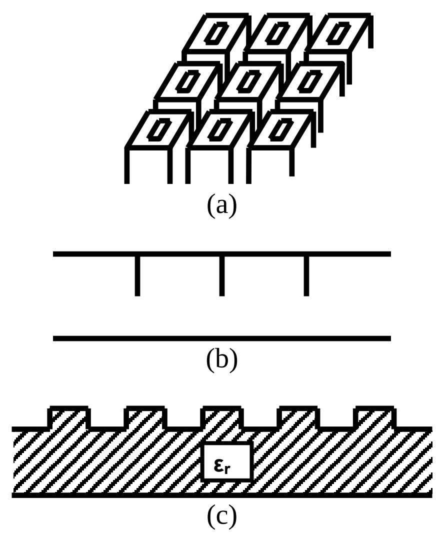

Fig.2 Typical periodic structures (a) 2-D open ended WG phased array, (b) Waveguide periodically loaded with fins, (c) Leaky wave antennas

TRANSMISSION LINE ANALYSIS

In order to understand the basic wave propagation phenomena associated with a periodic structure, let us use the simplest structure, namely Fig.1. Note that this periodic structure is made of infinite repetition of the unit cell. Unit cell consists of a transmission line of a length d at the center of which a lumped shunt admittance B (normalized with respect to the characteristic impedance of the line) is placed. If we consider a unit cell as a two port network, the so-called the ABCD matrix can be obtained by cascading three sections, namely one half (d/2) of the transmission line, jB and another half (d/2) of the transmission line. Therefore, the input and output relationship of the n-th unit cell is given by

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \qquad (1)$$

where

$$A = \cos\theta \ -(B/2)\sin\theta$$

$$B = j((B/2)\cos\theta + \sin\theta - B/2)$$

$$C = j((B/2)\cos\theta + \sin\theta + B/2)$$

$$D = \cos\theta \ -(B/2)\sin\theta$$

In the above $\theta = kd$ is the electrical length of the transmission line in the unit cell, k is the wave number and is equal to the propagation constant of this TEM line.

If we assume that the periodic structure is infinitely long, the wave phenomena must be identical at the input to the n-th cell and at the input to the (n+1)th cell, except for the phase delay caused by propagation along the cell. Therefore, it can be written that

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = e^{-\gamma d} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \qquad (2)$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant of the periodic structure. Since (1) and (2) are identical, the eigenvalue equation is obtained as follows.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} e^{-\gamma d} & 0 \\ 0 & e^{\gamma d} \end{bmatrix} \qquad (3)$$

The above can be reduced to

$$\cosh\gamma d = \cos\theta - (B/2)\sin\theta \qquad (4)$$

If the magnitude of the right hand side is smaller than unity, then $\alpha = 0$ and $\gamma = j\beta$. Hence, under this condition, the periodic structure supports a propagating wave. On the other hand, if the magnitude of the right hand side of (4) is larger than unity, then no wave can propagate along the structure.

Exercise: Derive the equation for α and for β.

From Fig.3 that plots the relationship (called the dispersion curve) between kd (proportional to frequency) and βd, the following two important characteristics are observed.

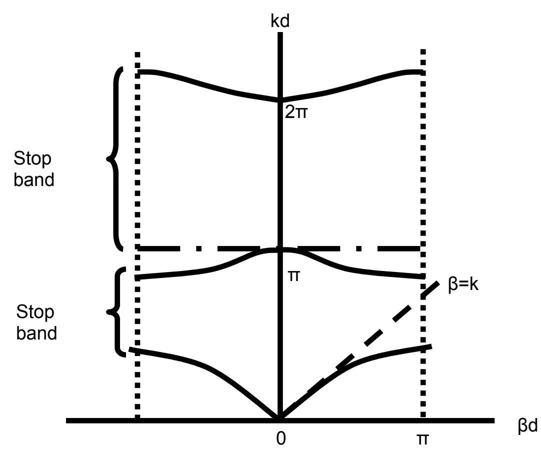

Fig.3 Typical dispersion curve of a non-radiating periodic structure

Passband and Stopband: These two situations of γ described above are called the passband and stopband. The frequency ranges corresponding to the solid lines in Fig.3 indicate the passband while the frequency regions without solid line correspond to the stopband.

Slow Wave Effect: In the absence of periodic lumped elements, the propagation constant along the structure is given by k. Namely, $\beta = \beta_0 = k$. For a given k or kd, the value of β or

βd on the dispersion curve of the periodic structure is always larger than β_0. Therefore, the phase velocity $v_p = kc/\beta$ (c: speed of light in vacuum) of the traveling wave in the periodic structure is slower than the free space speed of light. Hence, the periodic structure acts as a slow wave structure in its passband region. One important application is the delay line.

SPATIAL HARMONICS AND FLOQUET THEOREM

Let us expand the above simple description to more general electromagnetic system with a periodic configuration. We will analyze the electromagnetic fields associated with the structure shown in Fig.4 that is a dielectric waveguide with periodic perturbation along its axis. The structure chosen here is periodic only in the z direction for simplicity.

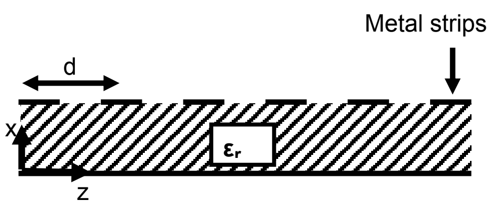

Fig.4 Dielectric waveguide with periodic perturbations along the axis

In this structure, any electromagnetic field components can be described by

$$\phi(x, y, z+d) = e^{-jk_{z0}d}\,\phi(x, y, z) \tag{5}$$

The exponential factor indicates the complex phase shift $k_{z0} = \beta - j\alpha$ between the neighboring unit cells. Therefore, the field differs only by this factor along the z direction. This expression is called Floquet theorem. The field component satisfying (5) can be described in the following manner.

$$\phi(x, y, z) = e^{-jk_{z0}z}\,P(x, y, z) \tag{6}$$

where P is a periodic function such that $P(x, y, z) = P(x, y, z+d)$. Therefore, P can be written as

$$P(x, y, z) = \sum_{n=-\infty}^{\infty} a_n(x, y)e^{-j\frac{2\pi n}{d}z} \tag{7}$$

where a_n is a function describing the field variations in the x and y directions and depends on the structure and excitation. From (6) and (7), the field can be written as

$$\phi(x, y, z) = \sum_{n=-\infty}^{\infty} a_n(x, y)e^{-jk_{zn}z} \tag{8}$$

where

$$k_{zn} = k_{z0} + \frac{2\pi}{d}n, \quad n = 0, \pm 1, \pm 2, \ldots. \tag{9}$$

The description in (8) is often called *Floquet spatial harmonic expansion.*

Instead of a rigorous analysis of the fields associated with the structure, let us first investigate what kind of dispersion diagram exists relating the frequency and the propagation factor (along z). For a further simplification of the problem, it is assumed that the structure is invariant in the y direction. In the absence of periodic perturbation, we recover a planar dielectric waveguide. A typical dispersion (k-β) diagram of the dominant guided mode is as shown in Fig.5. In this diagram there are four regions divided by the straight lines $k = \pm\beta$ (often called the air line). If the value of β for a given k is in Region I, the wave is guided along +z direction. Since $k < \beta$, the phase velocity is smaller than free space speed of light and the wave is called slow. If the dispersion curve gets into Region II as we will see later, then $k > \beta$ so that the wave is fast. To satisfy such a condition, the field in the region outside the dielectric region is no longer decaying in the $+x$ direction but rather becomes propagating type in $+x$. Regions III and Region IV correspond to Regions II and I except that β takes a negative value (and the wave is backward going).

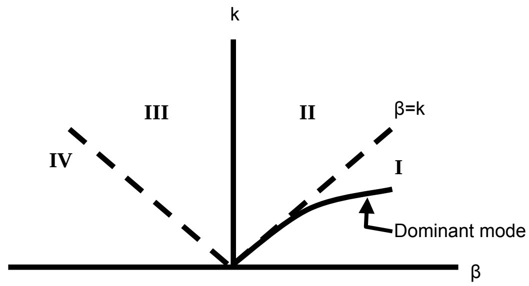

Fig.5 k-β diagram for an unperturbed dielectric waveguide

Let us now consider the case where individual perturbations are infinitesimally small. Hence, it is possible to introduce an approximation that $k_{z0} = \beta - j\alpha \approx \beta_0$ where β_0 is the propagation constant of the unperturbed dielectric waveguide. Nevertheless, the field associated with the periodic structure should be described by Floquet spatial harmonic expansion given by (8). Hence, $k_{zn} \approx \beta_n = \beta_0 + 2\pi n/d$, $n = 0, \pm 1, \pm 2, \ldots$. For each n, the relationship of β_n with k can be obtained by shifting the k-β diagram of the unperturbed structure to in the horizontal direction as shown in Fig.6.

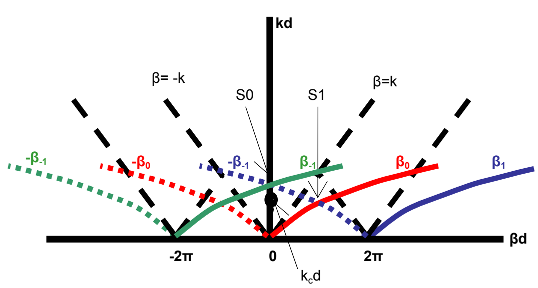

Fig.6 Spatial harmonic diagram normalized by the period d

This figure is a very useful one for understanding the wave behaviors associated with the (infinite) periodic structure. If the k value is below a certain value k_c, all spatial harmonics are in the slow wave region, if the mode of the unperturbed dielectric waveguide is guided. However, once k exceeds k_c due to an increased operating frequency, β_{-1} spatial harmonic is now in the fast wave region so that this periodic structure can now support the wave that is radiating away from the dielectric surface, or the dielectric waveguide is now used as a (**leaky wave**) antenna, even though the original waveguide is not leaky. The main beam direction of this antenna is determined by the frequency. If the curve β_{-1} intersect $\beta d = -kd$ (air line), then the beam is in the backward direction while the beam is forward endfire direction when the frequency is increased so that β_{-1} intersects the air line $\beta d = kd$. The direction of radiation measured from the normal of the waveguide surface is given by

$$\theta = \sin^{-1}(\beta_{-1}/k) \tag{10}$$

STOP BAND PHENOMENA (BANDGAP)

The above discussions are in the case where each perturbation is infinitesimally small. In practice, at each perturbation the wave is scattered. Hence, the dispersion diagram becomes more complex. First, it is possible that the propagation can be backward so that $-\beta_0$ and all of its spatial harmonics may appear as shown with broken lines in Fig.6. First, the $-\beta_{-1}$ curve indicates a backward wave meaning that the group velocity and the phase velocity is in the opposite direction for a positive β. Let us now pay attention to the point indicated by S1 in Fig.6. At this value of k or corresponding frequency, β_0 curve and $-\beta_{-1}$ curve intersect. Therefore, the wave coupling phenomenon takes place that is energy exchange between the waves with opposite group velocities. As a result, the dispersion curves are split as shown in Fig.7 where the solid lines are the outcome of the wave coupling. There is no real value solution of β between the two lines. No wave therefore can propagate along the axial direction of the waveguide in the region identified as the "stop band" in the figure. This corresponds to the case of $\beta = 0$ in (4). The shape and width of the stop band depend on the physical configuration of the periodic structure. An additional phenomenon is observed in the passband region below the stop band. Since the dispersion curve is pushed downward, the phase velocity becomes smaller. This is a **slow wave** phenomenon. Periodic structure is often used for a delay line device.

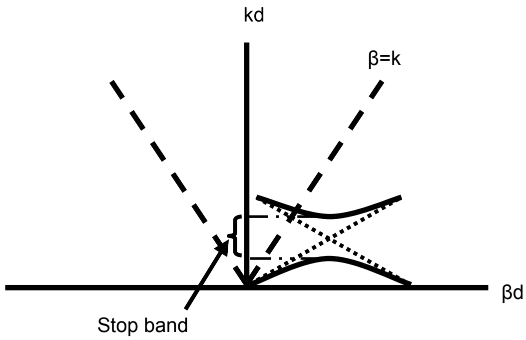

Fig.7 Stop band and slow wave phenomena

This stopband phenomenon is the basis of the so-called the Photonic Bandgap (PBG) structure of the Photonic Crystal although these are generally referred to as two-dimensional periodic structures.

The wave coupling also takes place at other intersection points in Fig.6. For instance, at S0 in Fig.6, forward and backward leaky waves are intersected and often create a leaky wave stop band. This phenomenon is quite complex. An immediate impact on the leaky wave antenna is its difficulty to radiate the beam in the broadside direction corresponding to the case of $\beta = 0$. Most leaky wave antennas try to avoid its use for the broadside radiation.

ELECTROMAGNETIC FIELD ANALYSIS

In this section, an excitation problem of a one dimensional periodic structure is introduced by using an example of a parallel plate phased array excited by an incoming plane wave. The solution makes use of Floquet formalism. Fig.8 shows the side view of a phased array made of infinitely many parallel plate waveguides ($z > 0$) extending to $z = +\infty$ (or in practice terminated by matched loads). The size of the waveguide is a and is infinite and invariable in the y direction. The waveguide wall thickness is assumed zero.

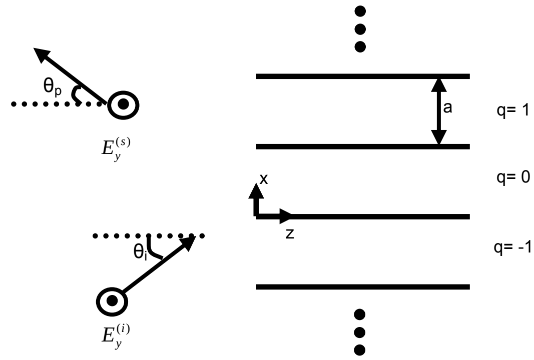

Fig.8 Infinite periodic array problem

An incident plane wave illuminates the waveguide array from $z < 0$ region with an incident angle of θ_i measured from the z axis. Since the structure is periodic, the scattered field in the region $z < 0$ can be expressed in terms of Floquet spatial harmonics.

$$E_y^{(s)}(x,z) = \sum_{p=-\infty}^{\infty} A_p \exp[-jkx\sin\theta_p + j\alpha_p z] \qquad z < 0 \qquad (11)$$

where

$$k\sin\theta_p = k\sin\theta_i + \frac{2p\pi}{a} \qquad p = 0, \pm1, \pm2, \cdots$$

$$\alpha_p = \sqrt{k^2 - (k\sin\theta_p + 2p\pi/a)^2}$$

$$= -j\sqrt{(k\sin\theta_p + 2p\pi a)^2 - k^2} \qquad (12)$$

Note that the incident field given by

$$E_y^{(i)}(x, z) = A\exp[-jkx\sin\theta_i - jkz\cos\theta_i] \qquad (13)$$

is actually periodic with the zeroth order corresponding to $p = 0$.

Inside the q-th waveguide

$$E_y(x,z) = \sum_{n=1}^{\infty} B_n^q \sin\left[\frac{n\pi}{a}(x - qa)\right]\exp(-j\beta_n z) \qquad z > 0$$

$$qa \le x < (q+1)a, \quad q = 0, \pm1, \cdots \qquad (14)$$

$$B_n^q = B_n^0 \exp(-jkaq\sin\theta_i)$$

Now, we match the total field $E_y^{(i)}(x,z) + E_y^{(s)}(x, z)$ and the corresponding magnetic field H_x with $E_y(x, z)$ and H_x in the waveguide region at $z = 0$. Notice that because of (14) we only need to match the fields only over one waveguide opening such as $0 \le x < a$. This process leads to an infinite set of linear equations with unknown A_p and B_n^q while A is specified as the intensity of the incoming wave.

Exercise: Drive the equations by way of the field matching at $z = 0$, $0 \le x < a$.

SOME REMARKS

Some of the interesting phenomena associated with the periodic structure are

1. **Stopband and Passband effects**
2. **Slow wave phenomena**
3. **Leaky wave phenomena**
4. **Backward wave phenomena**

A word of caution on the last item is given here. Recently, there is excitement in physics and electromagnetic field communities on the so-called **metamaterials** that are loosely interpreted as artificially synthesized material structures to realize characteristics not available in natural materials. Of these, the so-called Left Handed Material or Double Negative Materials (as effectively negative permittivity and negative permeability are realized simultaneously) provide many interesting wave phenomenal including the backward wave. However, this backward wave is different from those observed in the higher order spatial harmonics such as $-\beta_{-1}$. The backward wave for LHM is the phenomena appearing in the fundamental mode. Although no periodicity is required for LHM, often a periodic structure is used mainly for convenience of fabrication. In that case, the size of the period is much smaller than the operating wavelength. On the other hand, Photonic Crystal is operated with a period close to one half of the wavelength.

Let us add brief comments of metamaterials, or more specifically left handed materials. Although several approaches exist for realization of the LHM, one way appealing way is to use a "transmission" line approach that supports a backward wave.

Fig.9 shows a model of the transmission line by means of distributed series LΔz and shunt CΔz. If Δz is infinitesimally small, the dispersion curve is a straight line $\beta = k$. Now, if C and L are interchanged as shown in Fig.10, the dispersion curve is like the one in the figure. Not only is this curve dispersive, the directions of the phase velocity and the group velocity are opposite. This is a characteristic of the left handed transmission.

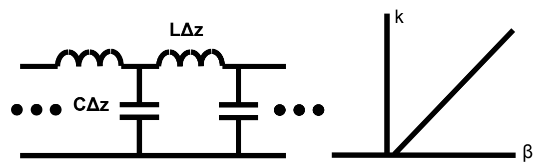

Fig.9 Distributed LC model for transmission line

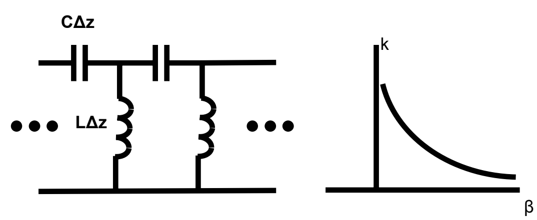

Fig.10 Distributed CL model of left handed line

Notice that if all L and C values are repetitive, both structures look like periodic structure. However, in the present case, the period given by Δ is infinitesimal. Therefore, the characteristics associated with periodic structures do not appear. Rather, the structure is called an effective medium. In fact, in order to synthesize the effective medium, it is not necessary to use periodic structures as long as the unit cell is much smaller than the operating wavelength.

CONCLUSIONS

In the above, a brief explanation of the fundamental aspects of periodic structures was presented. Physical insight is emphasized. To this end the topics are restricted to one-dimensional cases. Extension to higher dimensions not only makes equations more complicated but also make to problem vectorial in general.

FURTHER READING

Due to the nature of the lecture, no specific references are provided. However, many graduate level textbooks on electromagnetic theory may provide sufficient descriptions of the periodic structures. Some examples are:

R. E. Collin, Field Theory of Guided Waves, IEEE Press

D. Pozar, Microwave Engineering, Wiley.